# Interactions' modeling in economics

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Pratiques numériques en SHS
Séminaire de la MSH Val de Loire

# Outline

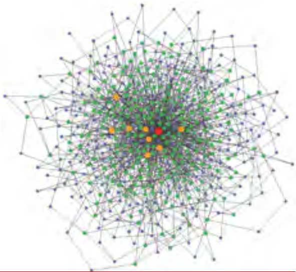
# Introduction

- Presentation of the INTERACT project
- Collaboration with Cem Ertur (LEO) and Daniel Mirza (LEO et GERCIE)

  Project over 3 ans financed by MSH Val de Loire
- Objective: Development of the visualisation tools regarding the results of the econometrics of interactions

# Defining Econometrics of Interactions

- What are the main characteristics of the world economy and finance ?



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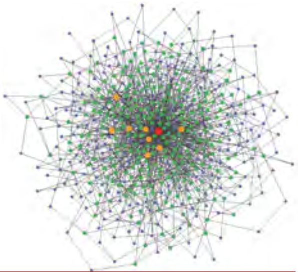

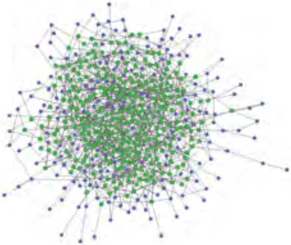

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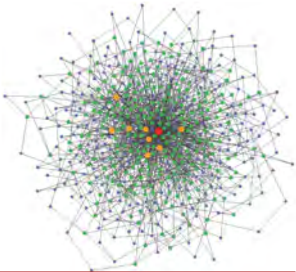
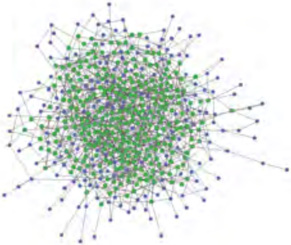
- What are the main characteristics of the world economy and finance ?
  - $\Rightarrow$ Globalization / cross-section interdependence / interactions
  - $\Rightarrow$ Networks
- How to include networks in econometric analysis ?
  - $\Rightarrow$ Graphs - Networks / Adjacency matrix $\Rightarrow$ Descriptive approach
  - $\Rightarrow$ Spatial Econometrics / Spatial Weight Matrix $\Rightarrow$ Explanatory approach





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  - ⇒ Spatial Econometrics / Spatial Weight Matrix ⇒ Explanatory approach
- Generalisation : Econometrics of interactions / Interaction matrix

# Example 1

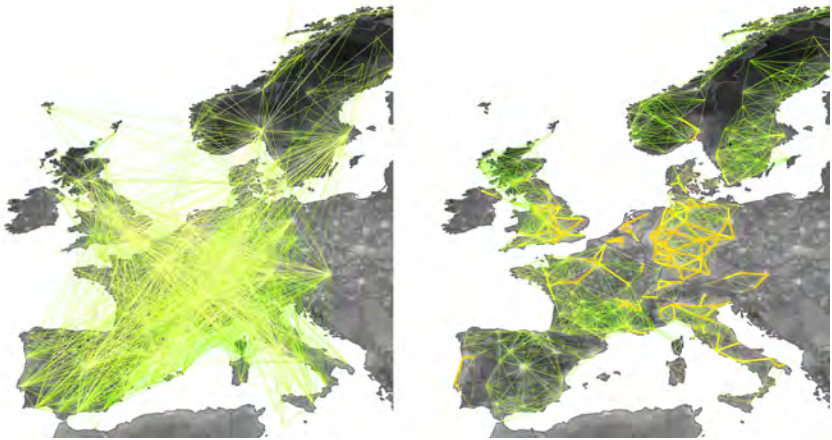

Figure: European mobility networks in epidemiology (GLEAMviz - Northeastern University)
Airport network (long range mobility) vs commuting network (short range mobility)

# Introduction

Objective of spatial econometrics

- Explicitly account for interactions in the specification of an economic model.

  Assume a non spatial model

$$y_i = \beta_1 + \beta_2 x_{2i} + \cdots + \beta_k x_{ki} + u_i$$
$$y_n = X_n \beta + u_n$$

  Only determinants of observation $i$ will affect its explained variable
- In many situations, assumption too strong:
  - Growth, peer effects, international trade, fiscal policies, Arabic spring

# Impacts in the classical regression model

- A simple cross-section model

$$y_i = \beta_1 + \beta_2 x_{2i} + \cdots + \beta_K x_{Ki} + u_i$$
$$u_i \sim i.i.d.(0, \sigma^2) \qquad i = 1, \ldots, N$$

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$\Rightarrow$ *No interaction / No spillover effect*

# Impacts in the classical regression model

- Stacking over all individuals $i = 1, ..., N$

$$\mathbf{y} = \beta_1 \iota + \beta_2 \mathbf{x}_2 + \cdots + \beta_K \mathbf{x}_k + \mathbf{u}$$

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- Impact matrices for $k = 1, ...K$

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}'_k} = \begin{pmatrix} \frac{\partial y_1}{\partial x_{1k}} & \dots & \frac{\partial y_1}{\partial x_{Nk}} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_N}{\partial x_{1k}} & \dots & \frac{\partial y_N}{\partial x_{Nk}} \end{pmatrix} = \begin{pmatrix} \beta_k & 0 & \dots & 0 \\ 0 & \beta_k & 0 \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & \beta_k \end{pmatrix} = \mathbf{I}_N \beta_k$$

Effets directs constants entre individus et absence d'effets indirects!

# SAR Model

Model traditionnally used to account for interactions between observations (SAR model):

$$y_n = \rho W_n y_n + X_n \beta + u_n$$

- $W_n$ specifies the relevant neighborhood for each observation $\Rightarrow$ Defines the interaction scheme / Matrix of interactions.
- What happens somewhere is partly determined by the same phenomenum in its neighborhood
- $\rho$ measures interactions' intensity between observations.
- $W_n y_n$ is the spatial lag.

# Interaction matrix

- Structuring spatial dependence

$$cov(y_i, y_j) \neq 0, \text{ for } i \neq j$$

- Which $i, j$ do interact?
  - $n$ observations to estimate $n(n-1)/2$ terms
    $\Rightarrow$ impossible
- Interaction hypotheses
  - $i$ and $j$ interact if they are "neighbors": similar / near
  - decreasing interaction with increasing dissimilarity / distance
- Definition of the relevant notion of space and neighborhood similarity-proximity / dissimilarity-distance measure $\Rightarrow$ interaction matrix $W$
- Should ideally be theory based
  see Behrens, Ertur and Koch (JAE, 2011) for an example in trade theory

# Interaction matrix

- Definition
  **W** is a $n \times n$ matrix with non-stochastic, non-negative, finite terms $w_{ij}$
  It must be exogenous with respect to the econometric model
- Simplest form: binary matrix
  $w_{ij} = 1$ if $i$ and $j$ are "neighbors"
  Examples:
  - $w_{ij} = 1$ if $i$ and $j$ are contiguous
  - $w_{ij} = 1$ if $d_{ij} < D$
  - $w_{ij} = 1$ if $i$ and $j$ are nearest neighbors

  $w_{ij} = 0$ otherwise and $w_{ii} = 0$ by convention
- Row-normalization
  $w^*_{ij} = \frac{w_{ij}}{\sum_{j \neq i} w_{ij}}$ so that $\sum_i w^*_{ij} = 1$.
  Allows interpretation of the spatial lag in terms of weighted average of the neighbors.

# Example 1

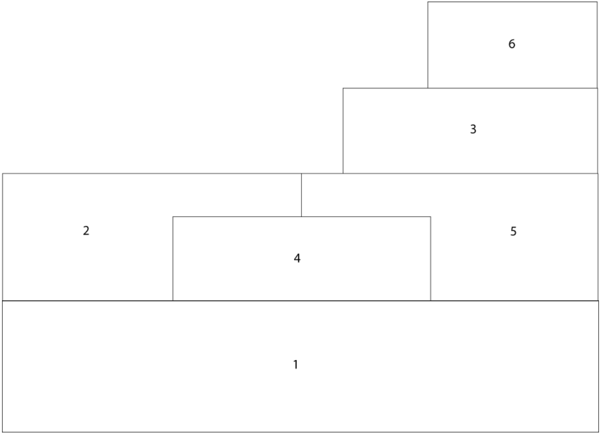



Figure: Map with $N = 6$ spatial units
Contiguity = common boundary / **W** is the spatial weight matrix

# Example 2

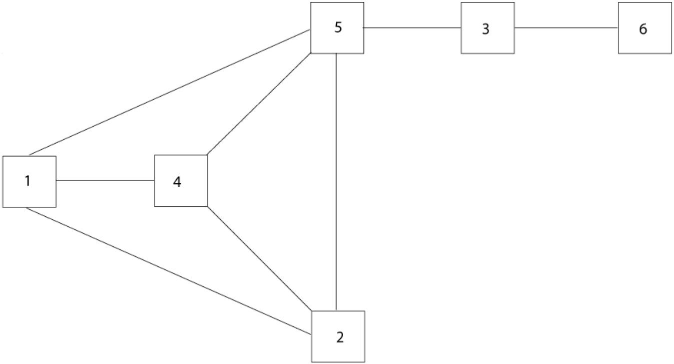



Figure: Undirected graph with $N = 6$ nodes
Contiguity = arcs between nodes / $\mathbf{W}$ is the adjacency matrix

# Example

- Row-normalized interaction matrix

$$\mathbf{W} = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{W}^* = \begin{pmatrix} 0 & 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 0 & 0 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

- Spatially lagged variable

$$\mathbf{W}^* y = \begin{pmatrix} 1/3y_2 + 1/3y_4 + 1/3y_5 \\ 1/3y_1 + 1/3y_4 + 1/3y_5 \\ 1/2y_5 + 1/2y_6 \\ 1/3y_1 + 1/3y_2 + 1/3y_5 \\ 1/4y_1 + 1/4y_2 + 1/4_3 + 1/4y_4 \\ y_3 \end{pmatrix}$$

# Outline

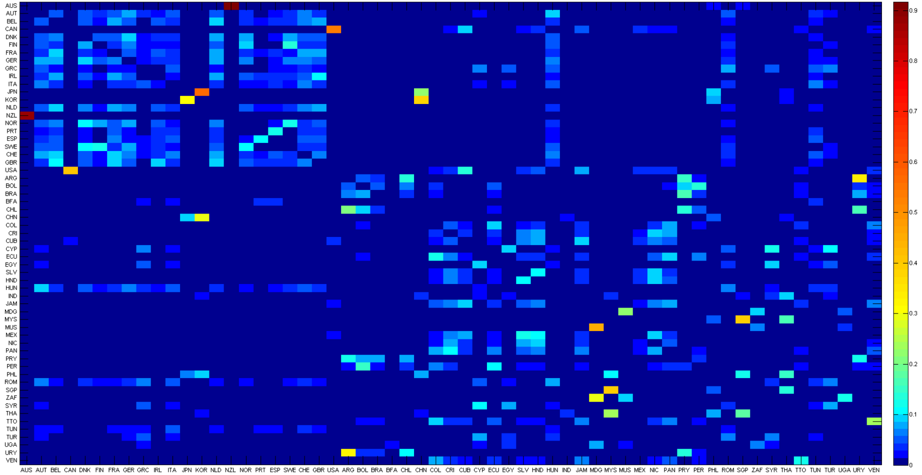
# Interactions matrices

- Matrices coming from Debarsy and Ertur (2014)
- **General term of the interaction matrices W** $= diag(H_i)\mathbf{V}$ where **V** is row-standardized.
  $H_i$, the human capital stock, is measured as suggested by Hall and Jones (1999) or Caselli (2005) based on the Mincerian wage equation
- **The geographic distance matrix** (**W1**)

$$w1_{ij} = H_i v1_{ij} \qquad v1_{ij} = \begin{cases} 0 \text{ if } & i = j \\ e^{-d_{ij}^2} / \sum_{j \neq i} e^{-d_{ij}^2} & \text{otherwise} \end{cases} \tag{1}$$

where $d_{ij}$ is great-circle geographic distance between country capitals.

# Geographic distance based normalized interaction matrix W1



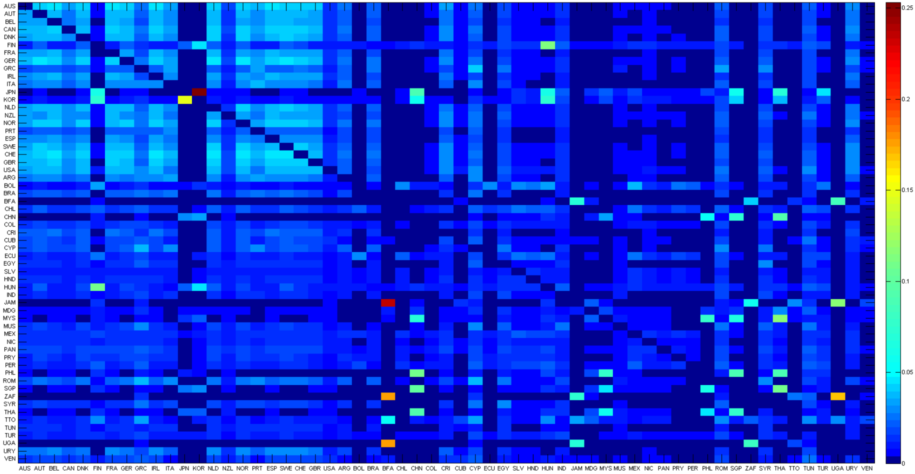
# Interactions matrices

- **The genetic distance matrix** (**W2**)

$$w2_{ij} = H_i v2_{ij} \qquad v2_{ij} = \begin{cases} 0 \text{ if } & i = j \\ e^{-2d_{ij}} / \sum_{j \neq i} e^{-2d_{ij}} & \text{otherwise} \end{cases} \qquad (2)$$

  where $d_{ij}$ is based on Spoalore and Wacziarg genetic distance (QJE, 2009).
- This notion of distance corresponds to the time elapsed since two populations shared a common ancestor: summary measure of relatedness between two populations.
- It is based on the differentiation coefficient $G_{ST}$ of Nei (1972) used in Population Genetics.
- A greater distance measure $G_{ST}$ reflects a longer separation between populations.
- However, as many countries, such as United States and Australia, are composed of sub-populations that are genetically distant, it is preferable to use a weighted genetic distance measure.

# Genetic distance based normalized interaction matrix W2



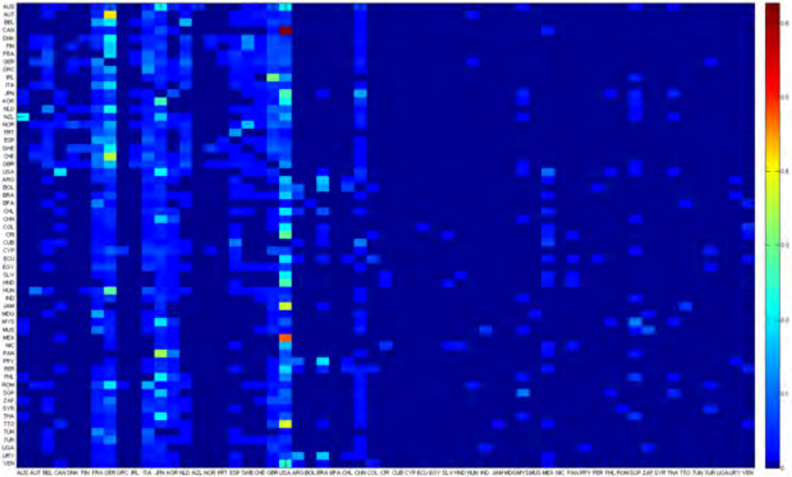
# Interactions matrices

- **The trade flows matrix (W3)**

$$w3_{ij} = H_i v3_{ij} \qquad v3_{ij} = \begin{cases} 0 \text{ if } \quad i = j \\ m_{ij} / \sum_{j \neq i} m_{ij} \quad \text{otherwise} \end{cases} \tag{3}$$

where $m_{ij}$ is defined as the average imports of country $i$ coming from country $j$ over the 1990-2000 period.

# Trade based normalized interaction matrix W3



# Interactions matrices

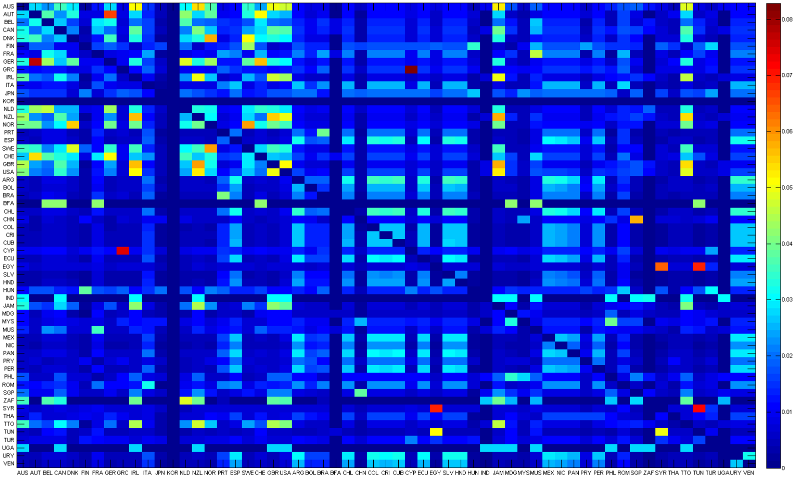
- **The linguistic proximity matrix (W4)**

$$w4_{ij} = H_i v4_{ij} \qquad v4_{ij} = \begin{cases} 0 \text{ if } & i = j \\ cl_{ij} / \sum_{j \neq i} cl_{ij} & \text{otherwise} \end{cases} \tag{4}$$

where $cl_{ij}$ represents the Common Language Index (CL) between countries $i$ and $j$ (Mélitz and Toubal, 2012).

- This index summarizes the evidence about the linguistic influences resting strictly on exogenous linguistic factors.
- This summary index (CL) is based on different proxies: Common Official Language (COL), Common Native Language (CNL) and Language Proximity (LP).

# Linguistic proximity based normalized interaction matrix W4



# Outline

# Impacts in the Spatial Autoregressive model

SAR Model: $\mathbf{y} = \rho \mathbf{W} \mathbf{y} + \mathbf{X}\beta + u$

- Explicit or reduced form under the suitable invertibility conditions on $\mathbf{I}_N - \rho \mathbf{W}$

$$\mathbf{y} = (\mathbf{I}_N - \rho \mathbf{W})^{-1} \iota_N \beta_0 + \sum_{k=1}^{K} (\mathbf{I}_N - \rho \mathbf{W})^{-1} \mathbf{I}_N \beta_k \mathbf{x}_k + (\mathbf{I}_N - \rho \mathbf{W})^{-1} u$$

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- Impact matrices for $k = 1, ..., K$

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}'_k} = (\mathbf{I}_N - \rho \mathbf{W})^{-1} \mathbf{I}_N \beta_k \neq \mathbf{I}_N \beta_k$$

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- where $(\mathbf{I}_N - \rho \mathbf{W})^{-1} = \sum_{r=0}^{\infty} \rho^r \mathbf{W}^r = \mathbf{I}_N + \rho \mathbf{W} + \rho^2 \mathbf{W}^2 + \cdots$ may in general be a full matrix

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# Impact matrices

- Let us define the $N \times N$ matrix $\mathbf{S}_k(\mathbf{W}) = (\mathbf{I}_N - \rho\mathbf{W})^{-1}\mathbf{I}_N\beta_k$
  $\mathbf{S}_k(\mathbf{W})$ is a full non symmetric matrix whose elements are:

$$\mathbf{S}_k(\mathbf{W}) = \begin{pmatrix} S_k(W)_{11} & S_k(W)_{12} & \dots & S_k(W)_{1N} \\ S_k(W)_{21} & S_k(W)_{22} & & S_k(W)_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ S_k(W)_{N1} & S_k(W)_{N2} & \dots & S_k(W)_{NN} \end{pmatrix}$$

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- The partial derivatives of $y_i$ relative to $x_{ik}$ or $x_{jk}$ for $i, j = 1, \dots, N$, $j \neq i$ and for $k = 1, \dots, K$ are then:

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- In general $S_k(\mathbf{W})_{ii} \neq S_k(\mathbf{W})_{jj}$ and $S_k(\mathbf{W})_{ij} \neq S_k(\mathbf{W})_{ji}$ for all $i, j$

# Impact Analysis

- Impact of a variation of $x_{ik}$ on $y_i$ (direct impact)

$$\frac{\partial y_i}{\partial x_{ik}} = S_k(\mathbf{W})_{ii} = (\mathbf{I}_N - \rho \mathbf{W})^{-1}_{ii} \beta_k \neq \beta_k$$
$$\frac{\partial y_i}{\partial x_{ik}} = S_k(\mathbf{W})_{ii} = (\mathbf{I}_N + \rho \mathbf{W} + \rho^2 \mathbf{W}^2 + \cdots)_{ii} \beta_k \neq \beta_k$$

$\mathbf{S}_k(\mathbf{W})_{ii}$: direct impact including feedback effects

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$\Rightarrow$ *Interactive heterogeneity*: differentiated direct impacts due to interactions between units

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$\mathbf{S}_k(\mathbf{W})_{ii}$: direct impact including feedback effects

⇒ *Interactive heterogeneity*: differentiated direct impacts due to interactions between units

- Impact of a variation of $x_{jk}$ on $y_i$ (indirect impact)

$$\frac{\partial y_i}{\partial x_{jk}} = S_k(\mathbf{W})_{ij} = (\mathbf{I}_N - \rho \mathbf{W})^{-1}_{ij} \beta_k \neq 0$$

⇒ The variation of an explanatory variable in $j$ will affect the dependent variable in $i$

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$$\frac{\partial y_i}{\partial x_{jk}} = S_k(\mathbf{W})_{ij} = (\mathbf{I}_N - \rho \mathbf{W})^{-1}_{ij} \beta_k \neq 0$$

$\Rightarrow$ The variation of an explanatory variable in $j$ will affect the dependent variable in $i$

$\Rightarrow$ *spillover effect*

# Example : Impact matrix

Table: Impact matrix, $N = 50$, $\beta = 1.84$ and $\rho = 0.65$

| | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
|---|---|---|---|---|---|---|---|
| 44 | **1.9462** | 0.0272 | 0.0621 | 0.0709 | 0.3035 | 0.1581 | 0.0020 |
| 45 | 0.0198 | **1.9838** | 0.0271 | 0.2713 | 0.0749 | 0.0052 | 0.0004 |
| 46 | 0.0223 | 0.0130 | **1.9708** | 0.0464 | 0.2557 | 0.0062 | 0.0055 |
| 47 | 0.0325 | 0.2333 | 0.0393 | **2.0013** | 0.1021 | 0.0079 | 0.0005 |
| 48 | 0.0581 | 0.0154 | 0.2225 | 0.0557 | **2.0275** | 0.0201 | 0.0028 |
| 49 | 0.2282 | 0.0272 | 0.0509 | 0.0708 | 0.2956 | **1.8952** | 0.0023 |
| 50 | 0.0041 | 0.0013 | 0.0516 | 0.0041 | 0.0359 | 0.0010 | **1.9170** |

# Example on the determinants of house price in Belgium

Assume the following model:

$$\mathbf{P}_t = \rho \mathbf{W}\mathbf{P}_t + \beta_0 + \beta_1 lincome_t + \beta_2 surf_t + \beta_3 \mathbf{W} surf_t + \beta_4 dens + \beta_5 env\_cha + \beta_6 pri\_fac + \beta_7 road\_qua + \sum_{p=1}^{9} PR_p + \mu + \varepsilon_t$$

Objectives

- Explain house price $\mathbf{P}_t$ by some determinants (income per capita, surface sold, population density, ...) for 588 Belgian municipalities
- Derive impact matrices
- Visualize impacts on a map.

# Outline

# Spatio-temporal models

Up to now, we did not consider dynamic in the model.
In Economics, phenomena generally are persistent over time (growth, development, labor norms,..)
$\Rightarrow$ Important to account for **space** and **time** dimensions

$$\mathbf{y}_t = \phi \mathbf{y}_{t-1} + \rho \mathbf{W}\mathbf{y}_t + \theta \mathbf{W}\mathbf{y}_{t-1} + \iota_N \alpha + \mu + \mathbf{x}_t \beta + + \mathbf{W}\gamma_r + \eta_t$$

Presence of 3 types of diffusion

- Temporal: $\mathbf{y}_{t-1}$
- Spatial: $\mathbf{W}\mathbf{y}_t$
- Spatio-temporal: $\mathbf{W}\mathbf{y}_{t-1}$

# Interpretation of the model

- A shock in a variable will last more than one period (due to temporal and spatio-temporal diffusions)
- 2 types of shocks: permanent vs transitory

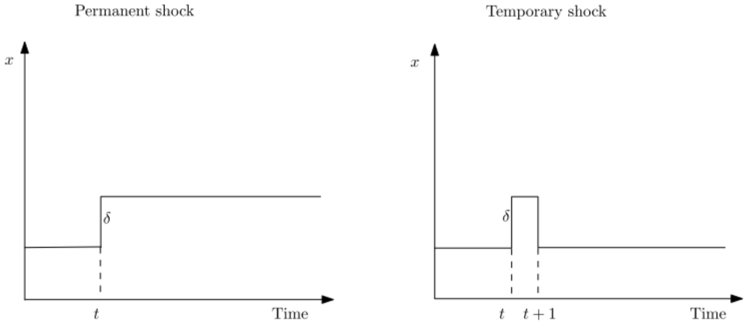


Figure: Comparison between permanent and transitory shocks

# Interpretation of the model

- Contemporaneous impact (purely spatial)

$$\partial \mathbf{Y}_t / \partial \mathbf{X}_t^{r'} = (\mathbf{I}_N - \rho \mathbf{W})^{-1} [\mathbf{I}_N \beta_r + \mathbf{W} \gamma_r] \quad (5)$$

⇒ Presence of direct and indirect effects

- T-period-ahead impacts

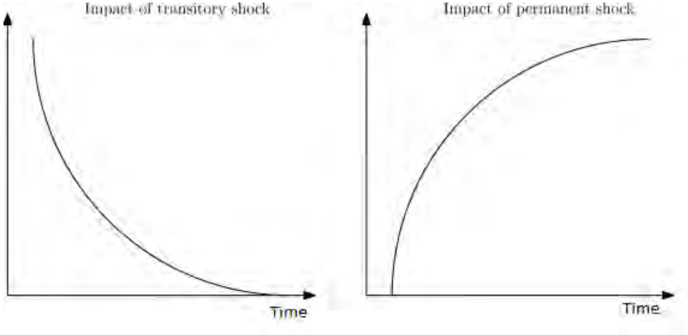


Figure: Comparison of time impacts due to transitory and permanent shocks

# Example on cigarette sales in US States

- Database of 45 (of the lower 48) states plus the District of Columbia from 1963-1992 taken (Baltagi and Li, 2004).
- Model is a simple (logged) demand equation for (packs of) cigarettes as a function of the (logged) cigarette prices (per pack) and (logged) state-level income per capita.
- Observations for 30 years on (logged) real per capita sales of cigarettes measured in packs per person aged 14 years or older (the dependent variable).
- The two explanatory variables are the (logged) average retail price of a pack of cigarettes and (logged) real per capita disposable income in each state and time period.
- log-log model. $\Rightarrow$ Impacts interpreted in terms of elasticities

# Estimation results

Table: Dynamic space-time model parameter estimates

| Parameters | lower 0.01 | lower 0.05 | mean | upper 0.95 | upper 0.99 |
|---|---|---|---|---|---|
| $\phi$ | 0.8099 | 0.8125 | 0.8326 | 0.8554 | 0.8584 |
| $\rho$ | 0.2825 | 0.2855 | 0.3040 | 0.3299 | 0.3333 |
| $\theta$ | -0.2801 | -0.2751 | -0.2511 | -0.2293 | -0.2211 |
| $\sigma_\mu^2$ | 0.0006 | 0.0007 | 0.0011 | 0.0018 | 0.0021 |
| $\sigma_\varepsilon^2$ | 0.0012 | 0.0012 | 0.0013 | 0.0014 | 0.0015 |
| | | | | | |
| Variables | lower 0.01 | lower 0.05 | mean | upper 0.95 | upper 0.99 |
| price | -0.3540 | -0.3406 | -0.2982 | -0.2555 | -0.2432 |
| income | 0.0348 | 0.0500 | 0.0989 | 0.1479 | 0.1645 |
| $W\times$ price | 0.1216 | 0.1376 | 0.1862 | 0.2323 | 0.2464 |
| $W\times$ income | -0.0883 | -0.0717 | -0.0206 | 0.0324 | 0.0492 |

Need to compute impacts to interpret the model (contemporaneous and space-time). Visual representation of the impacts

# Conclusions and future research

- Presentation of the econometrics of interactions' tools to explicitly account for the presence of interactions
- These tools can be used in other fields than economics
- Presentation of visual tools to help the interpretation of the model
- Need to work on the visualisation tools
  - Use of network analysis
  - Improve cartography tools and information provided